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Can the SO(10) Model with Two Higgs Doublets Reproduce the Observed Fermion Masses?

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It is usually considered that the SO(10) model with one **10** and one **126** Higgs scalars cannot reproduce the observed quark and charged lepton masses. Against this conventional conjecture, we find solutions of the parameters which can give the observed fermion mass spectra. The SO(10) model with one **10** and one **120** Higgs scalars is also discussed.

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I. INTRODUCTION

The grand unification theory (GUT) is very attractive as a unified description of the fundamental forces in the nature. Especially, the SO(10) model is the most attractive to us when we take the unification of the quarks and leptons into consideration. However, in order to reproduce the observed quark and lepton masses and mixings, usually, a lot of Higgs scalars are brought into the model. We think that the nature is simple. What is of the greatest interest to us is to know the minimum number of the Higgs scalars which can give the observed fermion mass spectra. A model with one Higgs scalar is obviously ruled out for the description of the realistic quark and lepton mass spectra. Then, how is a model with two different types of Higgs scalars (e.g., **10** and **126** scalars)?

In the SO(10) GUT scenario, a model with one **10** and one **126** Higgs scalars leads to the relation [1]

$$M_e = c_u M_u + c_d M_d, \tag{1.1}$$

where M_e, M_u and M_d are charged lepton, up-quark and down-quark mass matrices, respectively. It is widely accepted that there will be almost no solution of c_u and c_d which give the observed fermion mass spectra. The reason is as follows: We take a basis on which the up-quark mass matrix M_u is diagonal ($M_u = D_u$). Then, the relation (1.1) is expressed as

$$\widetilde{M}_e = c_u D_u + c_d \widetilde{M}_d. \tag{1.2}$$

Considering that $\widetilde{M}_d$ is almost diagonal and the mass hierarchy of up-quark sector is much severe than that of down-quark sector, we observe that the contribution to the first and the second generation part of $\widetilde{M}_e$ from the up-quark part D_u is negligible so that it is proportional to that of $\widetilde{M}_d$. Thus, the relation (1.1) which predicts $m_e/m_\mu \simeq m_d/m_s$ does not reproduce the observed hierarchical structure of the down-quark and charged lepton masses [2] such as predicted by Georgi-Jarlskog mass relations $m_b = m_\tau$, $m_s = m_\mu/3$ and $m_d = 3m_e$ at the GUT scale [3]. However, the above conclusion is somewhat impatient one. (i) It is too simplified to regard $\widetilde{M}_d$ as almost diagonal. (ii) We must check a possibility that the mass relations are satisfied with the opposite signs, i.e., $m_b = \pm m_\tau$, $m_s = \pm m_\mu/3$ and $m_d = \pm 3m_e$. (iii) The mass values at the GUT scale which are evaluated from the observed values by using the renormalization group equations show sizable deviations from the Georgi-Jarskog relations. The purpose of the present paper is to investigate systematically whether there are solutions of c_u and c_d which give the realistic quark and lepton masses or not.

II. OUTLINE OF THE INVESTIGATION

In the SO(10) GUT model with one **10** and one **126** Higgs scalars, the down-quark and down-lepton mass matrices M_d and M_e are given by

$$M_d = M_0 + M_1, \quad M_e = M_0 - 3M_1, \tag{2.1}$$

where M_0 and M_1 are mass matrices which are generated by the **10** and **126** Higgs scalars ϕ_{10} and ϕ_{126}, respectively. Inversely, we obtain

$$M_0 = \frac{1}{4}(3M_d + M_e), \quad M_1 = \frac{1}{4}(M_d - M_e). \tag{2.2}$$

On the other hand, the up-quark mass matrix M_u is given by

$$M_u = c_0 M_0 + c_1 M_1, \tag{2.3}$$

where

$$\begin{aligned} c_0 &= v_0^u/v_0^d = \langle\phi_{10}^{u0}\rangle/\langle\phi_{10}^{d0}\rangle, \\ c_1 &= v_1^u/v_1^d = \langle\phi_{126}^{u0}\rangle/\langle\phi_{126}^{d0}\rangle, \end{aligned} \tag{2.4}$$

and ϕ^u and ϕ^d denote Higgs scalar components which couple with up- and down-quark sectors, respectively. Therefore, by using the relations Eq.(2.2), we obtain the relation

$$M_e = c_d M_d + c_u M_u, \tag{2.5}$$

where

$$c_d = -\frac{3c_0 + c_1}{c_0 - c_1} \ , \quad c_u = \frac{4}{c_0 - c_1} \ . \tag{2.6}$$

For convenience, first, we investigate the case that the matrices M_u, M_d and M_e are symmetrical matrices at the unification scale because we assume that they are generated by the **10** and **126** Higgs. Then, we can diagonalize those by unitary matrices U_u, U_d and U_e, respectively, as

$$U_u^T M_u U_u = D_u \ , \quad U_d^T M_d U_d = D_d \ , \ U_e^T M_e U_e = D_e \ , \tag{2.7}$$

where D_u, D_d and D_e are diagonal matrices. Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix V is given by

$$V = U_u^T U_d^* \ , \tag{2.8}$$

the relation (2.5) is re-written as follows:

$$(U_e^\dagger U_u)^T D_e (U_e^\dagger U_u) = c_d V D_d V^T + c_u D_u . \tag{2.9}$$

At present, we have almost known the experimental values of D_e, D_u and $V D_d V^\dagger$. Therefore, we obtain the independent three equations:

$$\mathrm{Tr} D_e D_e^\dagger = |c_d|^2 \, \mathrm{Tr}\Big[(V D_d V^T + \kappa D_u)(V D_d V^T + \kappa D_u)^\dagger\Big], \tag{2.10}$$

$$\mathrm{Tr}(D_e D_e^\dagger)^2 = |c_d|^4 \, \mathrm{Tr}\Big[((V D_d V^T + \kappa D_u)(V D_d V^T + \kappa D_u)^\dagger)^2\Big], \tag{2.11}$$

$$\det D_e D_e^\dagger = |c_d|^6 \det\Big[(V D_d V^T + \kappa D_u)(V D_d V^T + \kappa D_u)^\dagger\Big], \tag{2.12}$$

where $\kappa = c_u / c_d$. By eliminating the parameter c_d, we have two equations for the parameter κ:

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^3}{m_e^2 m_\mu^2 m_\tau^2} = \frac{(2.10)^3}{(2.12)}, \tag{2.13}$$

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^2}{2(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2)} = \frac{(2.10)^2}{(2.10)^2 - (2.11)}, \tag{2.14}$$

where $(2.10)^3$, for instance, means the right-hand side of (2.10) to the third power. Let us denote the parameter values of κ evaluated from (2.13) and (2.14) as κ_A and κ_B, respectively. If κ_A and κ_B coincide with each other, then we have a possibility that the SO(10) GUT model can reproduce the observed quark and lepton mass spectra. If κ_A and κ_B do not so, the SO(10) model with one **10** and one **126** Higgs scalars is ruled out, and we must bring more Higgs scalars into the model. Of course, in the numerical evaluation, the values κ_A and κ_B will have sizable errors, because the observed values D_e, D_u, D_d and V have experimental errors, and the values at the GUT scale also have errors. The values κ_A and κ_B are not so sensitive to the renormalization group equation effect (evolution effect), because those are almost determined only by the mass ratios. (More details will be discussed in the Sec. III.) Therefore, we will evaluate κ_A and κ_B by using the center values at $\mu = m_Z$ in the Sec. IV. If we find $\kappa_A \simeq \kappa_B$, we will give further detailed numerical study only for the case.

III. EVOLUTION EFFECT

The relations (2.13) and (2.14) hold only at the unification scale $\mu = \Lambda_X$ On the other hand, we know only the experimental values of the fermion masses m_f and CKM matrix parameters V_{ij} at the electroweak scale $\mu = m_Z$. For a model which does not have any intermediate energy scales, we can straightforwardly estimate the values of m_f and V_{ij} at $\mu = \Lambda_X$ from those at $\mu = m_Z$ by the one-loop renormalization equation

$$\frac{dY_f}{dt} = \frac{1}{16\pi^2}(T_f - G_f + H_f) Y_f \tag{3.1}$$

where T_f, G_f and H_f denote contributions from fermion-loop corrections, vertex corrections due to the gauge bosons and vertex corrections due to the Higgs boson(s), respectively. Therefore, we can directly check the relations (2.13) and (2.14) by substituting the observable quantities m_f and V_{ij} at $\mu = \Lambda_X$. However, for a model which has an intermediate energy scale such as a non-SUSY model, the values of m_f and V_{ij} at $\mu = \Lambda_X$ are highly model-dependent, so that the check of Eqs. (2.13) and (2.14) cannot be done so straightforwardly.

In this section, we will show that we can approximately check Eq. (2.13) and (2.14) by using the values of m_f and V_{ij} at $\mu = m_Z$, without knowing the explicit values of m_f and V_{ij} at $\mu = \Lambda_X$, as far as the evolutions of m_f and V_{ij} are not singular.

It is well known that in such a conventional model the evolution effects are approximately described as [4]

$$\frac{m_u^0 / m_t^0}{m_u / m_t} \simeq \frac{m_c^0 / m_t^0}{m_c / m_t} \simeq 1 + \varepsilon_u \ ,$$

$$
\begin{aligned}
&\frac{m_d^0/m_b^0}{m_d/m_b} \simeq \frac{m_s^0/m_b^0}{m_s/m_b} \simeq 1+\varepsilon_d\ , \\
&\frac{|V_{ub}^0|}{|V_{ub}|} \simeq \frac{|V_{cb}^0|}{|V_{cb}|} \simeq \frac{|V_{td}^0|}{|V_{td}|} \simeq \frac{|V_{ts}^0|}{|V_{ts}|} \simeq 1+\varepsilon_d\ , \\
&\frac{m_u^0/m_c^0}{m_u/m_c} \simeq \frac{m_d^0/m_s^0}{m_d/m_s} \simeq \frac{|V_{us}^0|}{|V_{us}|} \simeq \frac{|V_{cd}^0|}{|V_{cd}|} \simeq 1\ ,
\end{aligned} \tag{3.2}
$$

where m_q^0 and V_{ij}^0 (m_q and V_{ij}) denote the values at $\mu=\Lambda_X$ ($\mu=m_Z$). The relations (3.2) hold only for a model where the Yukawa coupling constant of top quark, $y_t \equiv (Y_u)_{33}$, satisfies $y_t \gg (Y_d)_{ij}$ ($i,j=1,2,3$). The relations (3.2) also hold even in a model which has an intermediate energy scale Λ_I, because, for example, when we denote $(m_u/m_t)_{\mu=\Lambda_X}/(m_u/m_t)_{\mu=\Lambda_I}$ and $(m_u/m_t)_{\mu=\Lambda_I}/(m_u/m_t)_{\mu=m_Z}$ as $1+\varepsilon_{u1}$ and $1+\varepsilon_{u2}$, respectively, we can obtain $(m_u/m_t)_{\mu=\Lambda_X}/(m_u/m_t)_{\mu=m_Z} \simeq 1+\varepsilon_u$ with $\varepsilon_u=\varepsilon_{u1}+\varepsilon_{u2}$.

By using the approximate relations (3.2) the diagonalized up-quark mass matrix D_u^0 at $\mu=\Lambda_X$ is presented as

$$
\begin{aligned}
D_u^0 &= m_t^0 \begin{pmatrix} m_u^0/m_t^0 & 0 & 0 \\ 0 & m_c^0/m_t^0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&\simeq m_t^0 \begin{pmatrix} m_u/m_t & 0 & 0 \\ 0 & m_c/m_t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1+\varepsilon_u & 0 & 0 \\ 0 & 1+\varepsilon_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \frac{m_t^0}{m_t}(1+\varepsilon_u S) D_u,
\end{aligned} \tag{3.3}
$$

where

$$
S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ . \tag{3.4}
$$

Similarly, the matrix D_d^0 is given by

$$
D_d^0 \simeq \frac{m_b^0}{m_b}(1+\varepsilon_d S) D_d. \tag{3.5}
$$

The CKM matrix V^0 at $\mu=\Lambda_X$ is given by

$$
\begin{aligned}
V^0 &\simeq \begin{pmatrix} 1 & V_{us} & V_{ub}(1+\varepsilon_d) \\ V_{cd} & 1 & V_{cb}(1+\varepsilon_d) \\ V_{td}(1+\varepsilon_d) & V_{ts}(1+\varepsilon_d) & 1 \end{pmatrix} \\
&\simeq (\mathbf{1}+\varepsilon_d S_3) V (\mathbf{1}+\varepsilon_d S_3) - 2\varepsilon_d S_3\ ,
\end{aligned} \tag{3.6}
$$

where $S_3=\mathbf{1}-S$ and $\mathbf{1}$ is a 3×3 unit matrix. By using the relations (3.4) - (3.6), we can obtain the approximate expression

$$
V^0 D_d^0 V^{0T} \simeq \frac{m_b^0}{m_b}\left[(1+\varepsilon_d) V D_d V^T - \varepsilon_d m_b S_3\right], \tag{3.7}
$$

where we have used the observed hierarchical relations among the quark mass ratios and CKM matrix parameters. Therefore, the matrix $VD_dV^T+\kappa D_u$ in Eqs. (2.10)-(2.12) is given by

$$
\begin{aligned}
K^0 &\equiv V^0 D_d^0 V^{0T} + \kappa^0 D_u^0 \\
&\simeq (1+\varepsilon_d)\frac{m_b^0}{m_b}\big(V D_d V^T + \kappa D_u \\
&\qquad -\varepsilon_d m_b S_3 + \varepsilon_u \kappa D_u S\big)\ ,
\end{aligned} \tag{3.8}
$$

where

$$
\kappa = \frac{m_t^0/m_t}{m_b^0/m_b}\frac{\kappa^0}{1+\varepsilon_d}\ . \tag{3.9}
$$

Since the solutions κ are of the order of 10^{-2} as we show in the next section, we can neglect the term $\kappa D_u S$ compared with VD_dV^T (note that in order to neglect the component $(D_uS)_{11}$ it is essential that the sign of m_d/m_s is positive, because $(VD_dV^T)_{11} \simeq m_d + V_{us}^2 m_s$ and $V_{us}^2 \simeq |m_d/m_s|$). On the other hand, for such a small value of κ, the term m_bS_3 cannot be neglected compared with the term κD_u. However, for a small value of ε_d, we can find that the solutions κ are substantially not affected by the term $\varepsilon_d m_b S_3$. As a result, we obtain the approximate expression

$$
K^0 \simeq (1+\varepsilon_d)\frac{m_b^0}{m_b}\left(V D_d V^T + \kappa D_u\right). \tag{3.10}
$$

Therefore, Eq. (2.13) and (2.14) at $\mu=\Lambda_X$, i.e.,

$$
\frac{[(m_e^0)^2+(m_\mu^0)^2+(m_\tau^0)^2]^3}{(m_e^0)^2(m_\mu^0)^2(m_\tau^2)^2} = \frac{[\mathrm{Tr}(K^0K^{0\dagger}])]^3}{\det(K^0K^{0\dagger})}\ , \tag{3.11}
$$

$$
\begin{aligned}
&\frac{[(m_e^0)^2+(m_\mu^0)^2+(m_\tau^0)^2]^2}{2[(m_e^0)^2(m_\mu^0)^2+(m_\mu^0)^2(m_\tau^0)^2+(m_\tau^0)^2(m_e^0)^2]} \\
&= \frac{[\mathrm{Tr}(K^0K^{0\dagger})]^2}{[\mathrm{Tr}(K^0K^{0\dagger})]^2-\mathrm{Tr}(K^0K^{0\dagger})^2}\ ,
\end{aligned} \tag{3.12}
$$

are approximately replaced by the relations at $\mu=m_Z$:

$$
\frac{(m_e^2+m_\mu^2+m_\tau^2)^3}{m_e^2 m_\mu^2 m_\tau^2} = \frac{[\mathrm{Tr}(KK^\dagger)]^3}{\det(KK^\dagger)}\ , \tag{3.13}
$$

$$
\begin{aligned}
&\frac{(m_e^2+m_\mu^2+m_\tau^2)^2}{2(m_e^2m_\mu^2+m_\mu^2m_\tau^2+m_\tau^2m_e^2)} \\
&= \frac{[\mathrm{Tr}(KK^\dagger)]^2}{\mathrm{Tr}[(KK^\dagger)]^2-\mathrm{Tr}(KK^\dagger)^2}\ ,
\end{aligned} \tag{3.14}
$$

where

$$
K = V D_d V^\dagger + \kappa D_u\ , \tag{3.15}
$$

and κ is given by Eq. (3.9). This means that when we find the solution κ at $\mu=m_Z$, the solution at $\mu=\Lambda_X$ also exists, no matter whether the model is a SUSY one or a non-SUSY one. Then, we can obtain the value κ^0 at $\mu=\Lambda_X$ from the relation (3.9) with the solution κ at $\mu=m_Z$.

IV. NUMERICAL STUDY AT $\mu = M_Z$

As mentioned in the preceding section, if the solution κ exists at the energy scale $\mu = m_Z$, the one at $\mu = \Lambda_X$ also exists. Therefore, we investigate the relations (2.13) and (2.14) at $\mu = m_Z$. Note that Eqs.(2.13) and (2.14) are realized by GUT scale because Eq.(2.7) is broken at $\mu = m_Z$. In the present section, tentatively, we assume that the Yukawa coupling constant Y_{10} and Y_{126} at $\mu = m_Z$ keep their forms symmetrical, so that we can put the observed values D_u, D_d and V at $\mu = m_Z$ into the relations (2.13) and (2.14). For the fermion masses at $\mu = m_Z$, we use the following values: [5]

$$\begin{aligned} &m_t = 181 \pm 13 \text{ GeV}, && m_b = 3.00 \pm 0.11 \text{ GeV}, \\ &m_c = 677^{+56}_{-61} \text{ MeV}, && m_s = 93.4^{+11.8}_{-13.0} \text{ MeV}, \\ &m_u = 2.33^{+0.42}_{-0.45} \text{ MeV}, && m_d = 4.69^{+0.60}_{-0.66} \text{ MeV}, \\ &m_\tau = 1746.7 \pm 0.3 \text{ MeV}, \\ &m_\mu = 102.75138 \pm 0.00033 \text{ MeV}, \\ &m_e = 0.48684727 \pm 0.00000014 \text{ MeV}. \end{aligned} \tag{4.1}$$

The input values for the CKM matrix parameters have been taken as [6]

$$\begin{aligned} &\theta_{12} = 0.219 - 0.226, \quad \theta_{23} = 0.037 - 0.043, \\ &\theta_{13} = 0.002 - 0.005, \end{aligned} \tag{4.2}$$

where

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{4.3}$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The calculation has been performed allowing all the combinations of the quark mass signatures. Here it should be noted that, since m_u is much smaller than m_c and m_t, the difference of the sign of m_u scarcely makes a change of allowed regions. In this calculation, we have selected θ_{23} and δ as input parameters and m_s, c_d and κ as output parameters because the calculation is sensitive to these parameters. We give the numerical results in Fig 1. Here, except for m_s, θ_{23} and δ, we have adopted the center values of Eq.(4.1) as input values. Moving θ_{23} at intervals of 0.0005 rad and fixing $\delta = 60°$, we search the solutions where κ_A and κ_B become coincident. Our numerical analysis shows that the solutions exist in the combinations of Table I. In a table II, we show the nearest solution of m_s, θ_{23} and δ to the center values of Eq.(4.1).

In the following we perform data fitting for the case of top line of Table II. Eqs. (2.10)-(2.12) can constrain only the absolute value of c_d. The argument of the parameter c_d may be decided by taking neutrino sector into consideration in the future. For the time being, we set $c_d \equiv |c_d|e^{i\sigma} = e^{0.107i}$ so that c_0 becomes a real number:

$$c_0 = \frac{1 - c_d}{c_u} = 34.7, \tag{4.4}$$

$$c_1 = -\frac{3 + c_d}{c_u} = 101.8 - 10.8i. \tag{4.5}$$

In this case, the mass matrices in MeV are

$$M_0 = \frac{3VD_dV^T + c_d(\kappa D_u + VD_dV^T)}{4} = \begin{pmatrix} -12.4 - 0.7i & -23.0 - 1.8i & 9.6 - 13.2i \\ -23.0 - 1.8i & -91.5 - 3.9i & 194.0 + 10.5i \\ 9.6 - 13.2i & 194.0 + 10.5i & 1874.9 - 180.0i \end{pmatrix}, \tag{4.6}$$

$$M_1 = \frac{VD_dV^T - c_d(\kappa D_u + VD_dV^T)}{4} = \begin{pmatrix} 4.19 + 0.69i & 7.68 + 1.43i & -3.72 + 4.09i \\ 7.68 + 1.43i & 24.14 + 3.88i & -65.05 - 10.48i \\ -3.72 + 4.09i & -65.05 - 10.48i & 1119.67 + 179.98i \end{pmatrix}. \tag{4.7}$$

Here, using the condition $\sqrt{|v_0^u|^2 + |v_0^d|^2 + |v_1^u|^2 + |v_1^d|^2} =$ 246GeV, we can get VEV's as

$$v_0^d = \frac{246\,[\text{GeV}]}{\sqrt{(|c_0|^2 + 1) + (|c_1|^2 + 1)|\rho|^2}} \tag{4.8}$$

with $\rho \equiv v_1^d/v_0^d$. Then, the Yukawa couplings about **10** and **126** become

$$Y_{10} = \frac{M_0}{v_0^d}, \quad Y_{126} = \frac{M_1}{v_1^d}. \tag{4.9}$$

We consider that the model should be calculable perturbativly. We can see that every element of the Yukawa coupling constants (4.9) is smaller than one if we take a suitable value of $|\rho|$.

V. 10 AND 120

In the SO(10) GUT scenario, we can also discuss the model with one **10** and one **120** by the same method. The Yukawa couplings of **10** and **120** are symmetric and antisymmetric, respectively. If we consider a case that the Yukawa coupling constants of **10** are real and **120** pure imaginary, we can make them Hermitian, i.e., $Y_{10}^\dagger = Y_{10}$ and $Y_{120}^\dagger = Y_{120}$. Therefore, by considering the real vacuum expectation values v_{10} and v_{120}, we can obtain the Hermitian mass matrices M_u, M_d and M_e:

$$\begin{aligned} M_d &= M_0 + M_2, \quad M_e = M_0 - 3M_2, \\ M_u &= c_0 M_0 + c_2 M_2. \end{aligned} \tag{5.1}$$

Then, we can diagonalize those by unitary matrices U_u, U_d and U_e as

$$U_u^\dagger M_u U_u = D_u, \quad U_d^\dagger M_d U_d = D_d, \quad U_e^\dagger M_e U_e = D_e. \tag{5.2}$$

Since the CKM matrix V is given by

$$V = U_u^\dagger U_d \ , \tag{5.3}$$

the relation (5.1) is re-written as follows:

$$(U_u^\dagger U_e) D_e (U_u^\dagger U_e)^\dagger = c_d V D_d V^\dagger + c_u D_u. \tag{5.4}$$

As stated previously, we have almost known the experimental values of D_e, D_u and $V D_d V^\dagger$. Therefore, we obtain the independent three equations:

$$\mathrm{Tr} D_e = c_d [\mathrm{Tr} D_d + \kappa \mathrm{Tr} D_u], \tag{5.5}$$

$$\mathrm{Tr} D_e^2 = c_d^2 [\mathrm{Tr} D_d^2 + 2\kappa \mathrm{Tr}(D_u V D_d V^\dagger) + \kappa^2 \mathrm{Tr} D_u^2], \tag{5.6}$$

$$\det D_e = c_d^3 \det(V D_d V^\dagger + \kappa D_u), \tag{5.7}$$

where $\kappa = c_u/c_d$. For the parameter κ, we have two equations:

$$\begin{aligned} &\frac{m_e^2 + m_\mu^2 + m_\tau^2}{(m_e + m_\mu + m_\tau)^2} \\ &\quad = \frac{\mathrm{Tr} D_d^2 + 2\kappa \mathrm{Tr}(D_u V D_d V^\dagger) + \kappa^2 \mathrm{Tr} D_u^2}{(\mathrm{Tr} D_d + \kappa \mathrm{Tr} D_u)^2}, \end{aligned} \tag{5.8}$$

$$\frac{m_e m_\mu m_\tau}{(m_e + m_\mu + m_\tau)^3} = \frac{\det(V D_d V^\dagger + \kappa D_u)}{(\mathrm{Tr} D_d + \kappa \mathrm{Tr} D_u)^3}. \tag{5.9}$$

Eqs. (5.8) and (5.9) are more simple than Eqs. (2.13) and (2.14). c_d and κ are real since we have assumed the M_u, M_d and M_e to be Hermitian. So the calculation is easier than the case for **10** and **126**. The numerical results are listed in Table III-IV.

VI. SUMMARY AND DISCUSSION

In conclusion, we have investigated whether an SO(10) model with two Higgs scalars can reproduce the observed mass spectra of the up- and down-quark sectors and charged lepton sector or not. What is of great interest is to see whether we can find reasonable values of the parameters c_u and c_d which satisfy the SO(10) relation (2.5) or not. For the case with one **10** and one **126** scalars, in a parameter $\kappa = c_u/c_d$, we have obtained two equations (2.13) and (2.14) which hold at the unification scale $\mu = \Lambda_X$ and which are described in terms of the observable quantities (the fermion masses and CKM matrix parameters). We have sought for the solution of κ approximately by using the observed fermion masses and CKM matrix parameters at $\mu = m_Z$ instead of the observable quantities at $\mu = \Lambda_X$. Although we have found no solution for real κ, we have found four solutions for complex κ which satisfy Eqs. (2.13) and (2.14) within the experimental errors. Similarly, we have found four solutions for a model with one **10** and one **120** scalars. It should be worth while noting that the solutions in the latter model are real. The latter model is very attractive because the origin of the CP violation attributes only to the **120** scalar. In the both models, we can make the magnitudes of all the Yukawa coupling constants smaller than one, so that the models are safely calculable under the perturbation theory.

By the way, note that the numerical results are very sensitive to the values of m_s and θ_{23}. For numerical fittings, it is favor that the strange quark mass m_s is somewhat smaller than the center value $m_s = 93.4$ MeV which is quoted in Ref. [5].

Also note that the relative sign of m_d to m_s in each solution is positive, i.e, $m_d/m_s > 0$ as seen in Tables I and III. It is well known that a model with a texture $(M_d)_{11} = 0$ on the nearly diagonal basis of the up-quark mass matrix M_u leads to the relation $|V_{us}| = \sqrt{-m_d/m_s}$ [7], where the relative sign is negative, i.e., $m_d/m_s < 0$. On the contrary, we can conclude that in the SO(10) model with two Higgs scalars, we cannot adopt a model with the texture $(M_d)_{11} = 0$.

In the present paper, we have demonstrated that the unified description of the quark and charged lepton masses in the SO(10) model with two Higgs scalars is possible. However, we have not referred to the neutrino masses. Concerning this problem, Brahmachari and Mohapatra have recently showed that one **10** and one **126** model is incompatible with large ν_μ-ν_τ mixing angle [8]. Since there are many possibilities for neutrino mass generation mechanism, we are optimistic about this problem, too. Investigating for a question whether an SO(10) model with two Higgs scalars can give a unified description of quark and lepton masses including neutrino masses and mixings or not is our next big task.

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num.	(m_t, m_c, m_u)	(m_b, m_s, m_d)	(m_τ, m_μ, m_e)
(a)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ \pm\ \pm)$
(b)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ \pm\ \pm)$

TABLE I. The combinations of the signs of (m_t, m_c, m_u), (m_b, m_s, m_d) and (m_τ, m_μ, m_e). The notation (m_t, m_c, m_u) $= (+\ -\ +)$ denotes $m_t > 0$, $m_c < 0$ and $m_u > 0$. Eqs. (2.13) and (2.14) are not affected by the signs of charged leptons.

	Input		Output		
	$\lvert\theta_{23}\rvert$[rad]	$\delta[^\circ]$	m_s[MeV]	$\lvert c_d\rvert$	κ
(a)	0.0420	60.	76.3	3.15698	$-0.01928 - 0.00089i$
	0.0420	60.	76.3	3.03577	$-0.01937 - 0.00101i$
(b)	0.0420	60.	76.3	3.13307	$-0.01929 - 0.00092i$
	0.0420	60.	76.3	3.00558	$-0.01939 - 0.00105i$

TABLE II. Four sets of parameters giving good data fitting at $\mu = m_Z$ for one **10** and one **126** Higgs scalars. (a) and (b) correspond to the mass signatures in Table I, and the upper and lower lines do to the two intersections in Fig.1

num.	(m_t, m_c, m_u)	(m_b, m_s, m_d)	(m_τ, m_μ, m_e)
(a-1)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ +\ +)$
(a-2)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ +\ -)$
(b-1)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ +\ +)$
(b-2)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ +\ -)$

TABLE III. The combinations of the signs of (m_t, m_c, m_u), (m_b, m_s, m_d) and (m_τ, m_μ, m_e) for one **10** and one **120** Higgs scalars.

	Input		Output		
	$\lvert\theta_{23}\rvert$[rad]	$\delta[^\circ]$	m_s[MeV]	c_d	κ
(a-1)	0.0415	60.	79.551	0.05905	-0.01957
(a-2)	0.0415	60.	79.238	0.06124	-0.01942
(b-1)	0.0415	60.	79.673	0.05855	-0.01960
(b-2)	0.0415	60.	79.316	0.06080	-0.01945

TABLE IV. Four sets of parameters giving good data fitting at $\mu = m_Z$ for one **10** and one **120** Higgs scalars. (a-i) and (b-i) correspond to the mass signatures in Table III.

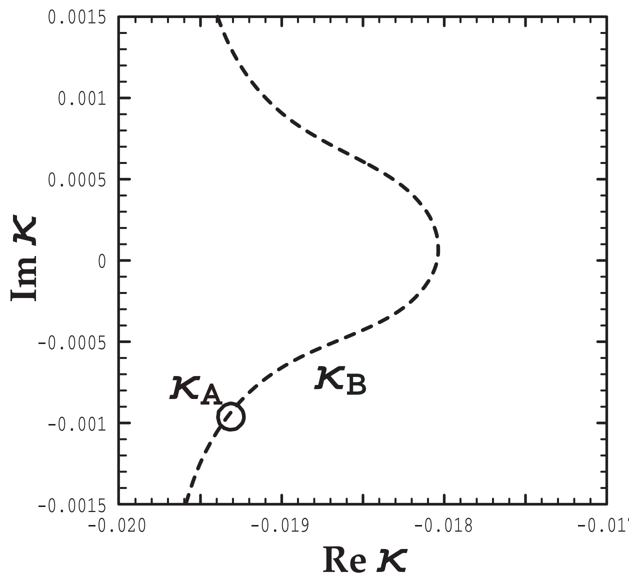

FIG. 1. The relations between Eqs.(2.13) and (2.14) on the complex plane of κ. The solid (dotted) line shows the solution of Eq.(2.13) (Eq.(2.14)).